AP Calculus AB
Dr. Paul L. Bailey

Activity 0128
Tuesday, January 28, 2024

Name:

Problem 1. Let $A = [0, 1]$ and $B = [0, 2]$. Find a function $f : A \to B$ such that:

(a) f is injective but not surjective

(b) f is surjective but not injective

(c) f is bijective

(d) f is neither injective nor surjective

Problem 2. Find a function $f : [0, \infty) \to [0, \infty)$ such that:

(a) f is injective but not surjective

(b) f is surjective but not injective

(c) f is bijective

(d) f is neither injective nor surjective

Problem 3. Let

$$f(x) = x^2 - 4x - 5.$$

Let $X \subset \mathbb{R}$ be the largest subset of $\mathbb{R}$ on which f is increasing. Let $Y = \{y \in \mathbb{R} \mid y = f(x) \text{ for some } x \in X\}$ be the image of X under f. Restrict f so that it is a function $f : X \to Y$.

(a) Write f in shifted form by completing the square.

(b) Write X and Y using interval notation.

(c) Explain why $f : X \to Y$ is bijective.

(d) Find a formula for $f^{-1} : Y \to X$.

(e) Sketch the graphs of f and f^{-1}.